The Eurasia Proceedings of
Educational & Social Sciences (EPESS)
ISSN: 2587-1730

The Eurasia Proceedings of Educational & Social Sciences (EPESS), XXXX

Volume XX, Pages XXX-XXX

ICRES 2019: International Conference on Research in Education and Science

Towards the design of a didactic engineering relying on economy as a semiotic model of mathematics

Pierre JOB

Ichec Brussels Management School

Jean-Yves GANTOIS

Ichec Brussels Management School

Abstract: Teaching mathematics, in a business and management school, led us to investigate the use of economy as a way to give alternative meaning to mathematical concepts, whether these concepts were already known to students - mostly from secondary school - or not. The intent behind this idea was to reverse the traditional connection between mathematics and economy. We tried to bridge the increasing gap between what students coming from secondary school are really capable of, irrespective of their grades, and what is expected from them in a business and management school. Instead of mathematics seen as a tool for economy, where mathematical theories are simply "applied" to economy, we developed an engineering where economy is used as a semiotic model of mathematics. Therefore, mathematics is not "applied" anymore to economy, it rather becomes consubstantial to economy, both disciplines acting on each other. This paper presents the structure of this engineering, the underlying hypothesis and the didactic motivations behind it.

Keywords: Engineering, Semiotic model, Economy, Mathematics, Transition

Introduction

This paper presents a didactic engineering designed for first year students entering a university level cursus in economy, business and management. This engineering hasn't been tested yet. The focus will be on exposing and justifying one of its principle, the use of economy as a semiotic model of mathematics. To carry out this plan we first need to explain the context in which this engineering started to grow.

Context

We are teaching in a Belgian high-school (university level) in economy, business and management and in charge of the same mathematical course given to 600 first year students, for more than six years. This course is the first at university level for most of our students. It underwent many changes over the years. Not for the sake of change but because of high failure rates. We had to face failure rates between 50% and 75% for the last six years. This

- Selection and peer-review under responsibility of the Organizing Committee of the Conference

was unacceptable for us and thus tried to reduce them using many different measures. Some measures were designed at the level of our institution or even by the minister of education. For instance:

- methodology courses are part of the curriculum (students learn how to study, take notes, …);
- a student only needs 10 out of 20 to pass an exam;
- under some circumstances, a student is allowed to access the following year without passing one or more exam, he can still pass them in the following years.

Others measures were devised over the years by the teachers in charge of the course. This means us and other colleagues. We are currently the only teachers in charge but it hasn't always been the case in the past (see below). Here are a few examples of "inside" measures.

- Decreased number of students in groups.
- Up to 5 teachers in charge of the course, thus more teachers per student.
- Less theory and more exercises.
- Course mostly stripped down from proof/demonstrations.
- Only a few theorems left.
- Detailed solutions to exercises to help students understand the way we want them to write down solutions.
- New exercises very similar to previous ones.
- Bonus points when doing some exercises.
- Improved explanation based on recurring errors.
- Exams based on questions solved almost "as is" during the course.
- Exam questions simplified over the years.
- A few weeks before the exam, preparation given to students, very similar to exam questions.

This is only a sample of the many directions we investigated. One measure that played a special role for us is the "reminder strategy". At some point, the course content was almost entirely based on reminders from secondary school: almost no new concepts were introduced. At that time, we choose to do so because we had to face the fact that our students didn't have the most basic skills, we taught we required to follow our course. For instance, many of them have trouble simply adding fractions, or computing the slope of a line. What kind of mathematics can you do when the most basic requirements are not fulfilled and the idea you have of mathematics is to teach within the framework of a deductive architecture, where knowledge is built onto one another? All these measures didn't seem to have much impact: failure rates remained the same. This gave rise to a deep questioning of the way we envisioned the teaching of mathematics and most notably of the reminders strategy that we taught was unavoidable. The first question we addressed was why were all these measures ineffective, especially the reminders one?

The platonic-formalist epistemology

We dug into the subject and found no easy answer to that question. Only a complex web of causes. We will not detail all these causes. Rather, we will single out one of them, that led us to adopt the principle underlying our engineering. This cause is the weight high-level cultural/epistemological constraints bear on the possible shape a mathematical course can take on (Chevallard,1992). In Job & Gantois (2017), we investigated the idea that « hidden » cultural/epistemological constraints deeply impact the possible structure of a mathematical course. In Belgium, many teachers have been raised in a specific epistemology: the platonic/formalist one. Roughly speaking, this epistemology relies on two postulates. The first one is that mathematical concepts reside somewhere in "mathematical heaven". The only way to get in touch with these concepts is through intuition. You either have it (mathematical intuition) or not. However fruitful, intuition can nevertheless be misleading. This leads to a second postulate. Mathematics have to be cast into a strictly deductive mould whose aim is to avoid the pitfalls of intuition. It means that, in the end, deductive reasoning is the only really relevant level of rationality that can be used to convey genuine mathematics.

Consequences on the educational system

This epistemology has deep consequences on the educational system. Many teachers are at a loss when facing students that do not remember/understand key concepts. Being driven by the deductive architecture of mathematics they tend, at a local level, to repeat over and over again the same bribe of deductive reasoning. This leads, when passing from one class to the next, to the reminders strategy. If a student doesn't understand mathematics at some point it must be because he doesn't understand or recall past pieces of the deductive architecture, those pieces he needs to rely on, at this stage of the learning process. Understanding mathematics is reduced to the way they are expressed in a deductive way (Bourbakism). For many teachers, the possibility of understanding functions of multiple variables without going through the theory of one-variable functions is dubious. They are at a loss when they have to face the fact that some students are able to pass an exam about very "abstract" concepts like the theory of categories when they struggle to pass a first-year exam dealing with limits, integrals, … When the reminders/repeating strategy fails (and it does), teachers are sort of forced into relying on the heavy use of charts, graphics, gestures to make their point. This process can go so far that the meaning of mathematics gets lost in the process (Brousseau, 2002). Teachers have a hard time envisioning the design of a course based on a different level of rationality than the deductive one (Schneider, 2010). This leads to failure like the counter-reformation of mathematics with mathematical courses mostly stripped down from their deductive architecture with not much else to replace it (Rouy, 2007).

Reducing economy to mathematics

Another consequence of the platonic/formalist epistemology is that economy (and other sciences) are often reduced to an application of mathematics. Being mathematized becomes a criterion of scientificness. We won't debate about the soundness of that criteria and focus on the didactic aspect. Let us simply note that not all economists agree with that kind of subordination (de Vroey, 2002). One major experimental drawback of reducing economy to mathematics is that students do not understand the link between economy and mathematics when "applying" mathematics to economy. For instance, simply stating, as an application of the theory of lines, that $2x + 5y = 100$ is the budget line of two goods with unit costs of 2 and 5 euros given a budget of 100 euros, x and y being the respective quantities of the goods that can be bought, is so obvious, that it becomes meaningless for many students. They do not understand why so much credit is given to budget lines. There is no problem to which the "budget line" would constitue an instrumental answer. From a certain angle the budget line is deprived of economic meaning. It is just a line that has been coloured with economic painting. We feel there is a sharp difference between a mathematical course filled with economic references and one which really relies on economy to build mathematics on top. Despite its economic labelling, the budget line lacks epistemological density.

Relying on economy

In contrast, in Job & Gantois (2018), we related the conclusion of a promising experience. Students were able to give meaning to inequalities like $ax + by \leq c$ using an economic context and relate them to half planes, again using economy as a semiotic guide. This conclusion is of great importance because, on the contrary, trying to teach them that $ax + by \leq c$ could be seen as a half-plane, in the sole setting of mathematics, failed for years. This experimental background led us to devise an engineering that would get rid of reminders and, instead, be structured with the idea that it is feasible to build mathematical concepts, in a way meaningful for students, starting from economic problems, and giving these concepts meaning through economy. In other words, we have argued the soundness of using "economy as a semiotic model of mathematics". Our arguments are based on experimental facts structured by an epistemological model (the platonic-formalist epistemology). Let us now turn to the structure of our engineering and see how the principle "economy as a semiotic model" shapes it.

A course centered around techniques and classes of problems

Reminders have been replaced by economic optimisation problems which form the core of the engineering. It should be stressed that we are not simply building a course around (more or less isolated) problems. In this sense, our approach is not a generic problem-based engineering, where we solely rely on "solving problems and everything else will come out naturally". These problems have a specific structure. They belong to the same class of problems we shall call C. This class is composed of problems asking to optimise a linear function of two variables, subjected to one linear constraint. Students are given problems taken from C but without being explicitly told that these can be considered as instances of the same class. They are simply faced with problems to solve. Moreover, these problems are not given in a formal way: there is no mention of linear functions or any formal object whatsoever. All these problems are expressed in casual language. Here is one example. Every other problem goes along the same lines.

- Two goods A_1 and A_2 are given.
- A_1 is sold at a price of 10€ per unit.
- A_2 is sold at a price of 15€ per unit.
- A_1 requires per unit the use of 2 units of another good U.
- A_2 requires per unit the use of 5 units of U.
- The quantity of U at hand is limited to 523 units.
- Which quantity of each A_1 and A_2 will maximize the profit subject to the limitation on U?

These problems have been chosen so that economy can be used to solve them. It means that no mathematical technique is forced upon the students right from the start. Enough problems of C are provided to students so they can device their own technique T to solve them. To students, these problems might not be as trivial as it may appear to the reader. Devising one single technique is expected to happen as a process. Let us briefly develop this aspect. One technique T_1 that should be expected from students is to simply produce as much as possible of the good that has the highest price. This technique is working on some instances, but not on every instance, as is the case of the example given above. Indeed A_1only uses 2 units of U compared to 5 units for A_2, it means that the each unit of U invested in A_1 has a return of $\frac{10}{2} = 5$ euros, whereas A_2 only has a return of $\frac{15}{5} = 3$ euros. Thus, producing as much as possible of the good having the lowest price is more profitable in this case. This can serve as the basis for another technique T_2. Moreover, the question of using entirely or not the quantity c of U at hand is not completely trivial. For instance, if the goods you are selling are rings, you cannot always use all of c simply because you cannot sell a fraction of a ring. This peculiar case may also bring about another technique T_3 that takes this specificity into account. Other aspects might be disturbing for the students but only an experiment could settle the matter. What we want to emphasize here is that these problems being new to students, up to a certain point, each new instance is susceptible to require a new technique to be solved or at least may require to amend an existing one. So, at first, the students will more likely develop a set of techniques T_1, T_2, …, T_k to solve the problems. Only when enough of these problems are given, will they have the opportunity to get rid of less efficient techniques and gradually build a more powerful one, T, integrating the different aspects of the problems. We can reasonably expect the following T to emerge from the various problems studied by the students. You choose to produce as much as possible of the good that has the greatest profitability i.e. the good which gives you the greatest benefit for each unit of U used in building that good. Then, if there are still units of U left, you complete your production with as much units as possible of the other good. Let us note that T is very much rooted in economic with its profitability concept. This anchoring into economy allows students to justify technique T and how sound it can be without being experts in "traditional" mathematical proof: there is no need for a sophisticated formal proof. This can be considered as the first step towards students edifying a theory. As even more problems are given to students, the question "Do you want to endlessly repeat the same calculations for each new problem or do you want to solve them, once and for all, using a general model of the problems?" will become an interesting issue, that will put them in a position where it becomes meaningful for them, to design a model of the problems they are given, on which they could apply T to solve all instances at once. In this case, the model will simply consist in replacing the various objects of the problems by letters, each having a specific status: variables, constants, unknown, …

- Two goods A_1 and A_2 are given.
- A_1 is sold at a price of p_1€ per unit.
- A_2 is sold at a price of p_2€ per unit.
- A_1 requires per unit the use of u_1 units of another good U.
- A_2 requires per unit the use of u_2 units of U.
- The quantity of U at hand is limited to c units.
- Which quantity q_1 and q_2 of each A_1 and A_2 will maximize the profit subject to the limitation on U?

In this model

- A_1, A_2 are just labels and not numbers, they are not constants nor unknowns nor anything of that sort.
- p_1, p_2, u_1, u_2, c are constants
- q_1, q_2 are unknowns

This is where C comes into explicit existence for the students, under the guise of an algebraic model and not just as a collection of problems that implicitly belong to the same class. This step will most likely be no small business for our students, as most of them struggle with letters and even more when it comes to distinguish between the various status of a letter. Once again, economy can be used to clarify the various status, based on previous numerical instances of the model. Thus, from the student point of view, qualifying the above model as a model is plainly relevant as it requires a conceptual effort to uses letters the proper way. The emergence of C then allows the students to apply T to the "abstract" algebraic model to solve all the problems falling under that model. Applying T to solve the model requires to express T in the language of the model and calls for a recast like the following. Let us define the profitability r_i of good A_i as the ratio $\frac{p_i}{u_i}$ and q_i^* the amount of A_i that maximizes the profit. Given those notations, T can be expressed as follows. If $r_i \geq r_j$, produce as much as possible of A_i and then as much as possible of A_j with the possible remains, that is $q_i^* = \left\lfloor \frac{c}{u_i} \right\rfloor$ and $q_j^* = \left\lfloor \frac{c - q_i^* u_i}{u_j} \right\rfloor$. This last stage constitutes a new step towards the creation of a theory. Students now have at their disposal an algorithm to solve problems from C and the justification to it. To sum up, C and T emerge in a dialectic process, each one acting on the other, those interactions being regulated through economic means.

Conclusion

We have presented, explained and justified a principle on top of which a didactic engineering, aimed at first year students, in a university level school of business, economy and management, is built. This principle amounts to using economy to give meaning and develop mathematical concepts useful in solving economic problems. The idea underlying this principle is to reverse the somewhat common scheme of "economy as a mere application of mathematics" into "mathematics forged in the crucible of economy". This idea emerged from our observations in classrooms and thus stems from an experimental background. Those observations were allowed by the epistemological reading grid, the platonic-formalist epistemology, we developed studying the reasons why measures taken to counteract high failures rates were ineffective. Thus, the principle and structure of our engineering relies on a didactic model rooted both in experimentation and theoretical epistemology. Given this cross-breeding nature, it took us quite some time to develop this engineering. We haven't been able to test it in classrooms yet. Further experimentations are required to inform us about the ecological viability of our engineering and the soundness of the underlying hypothesis. We intend to do so in the coming years taking all the necessary precautions. Indeed, another reason worth mentioning is that our engineering is a rather large scale one. It is designed to fit an entire course. From an institutional perspective, we cannot afford to turn it into a complete failure with students not diving into it. To us this makes a huge difference with respect to engineering that were "only" experimented at a much smaller scale with not much impact on the global ecology of a course. In other words, it is a much less risky business to experiment an engineering on a few selected students outside the beaten path of regular courses than an engineering impacting 600 first year students for their first course at university level.

References

Brousseau, G. (2002), *Theory of Didactical Situations in Mathematics*, Netherlands: Springer.

Chevallard, Y. (1992). Concepts fondamentaux de la didactique : perspectives apportées par une approche anthropologique. *Recherches en Didactique des Mathématiques*, 12(1), 73-112.

de Vroey, M. (2002). La mathématisation de la théorie économique : le point de vue de l'histoire des théories économiques. *Reflets et perspectives de la vie économique*, 4(41), 9-20.

Job, P., & Gantois, J.-Y. (2017). High level constraints weighting on the possible shapes knowledge can take on. In M. Shelley M., M. Pehlivan (Ed.), *Education Research Highlights in Mathematics in Science and Technology 2017* (pp. 14-20). Ames, IA: ISRES Publishing.

Job, P. and Gantois, J.-Y. (2018) Investigating economy as a mean to rejuvenate fossilized mathematical knowledge. The Eurasia Proceedings of Educational & Social Sciences (EPESS), 10, 248-253.

Rouy, J. (2007). *Formation initiale des professeurs de l'enseignement secondaire supérieur et changements de rationalité mathématique entre l'institution secondaire et l'institution universitaire. Le cas éclairant du thème des dérivées* (Doctoral dissertation). Université de Liège, Liège.

Schneider, M. (2010), Traité de didactique des mathématiques : la didactique par des exemples et contre-exemples, Liège : Presses Universitaires de Liège.

Author Information	
Pierre Job	Jean-Yves Gantois
Ichec Brussels Management School	Ichec Brussels Management School
Boulevard Brand Whitlock 4,	Boulevard Brand Whitlock 4,
1150 Woluwe-Saint-Pierre	1150 Woluwe-Saint-Pierre
Belgique	Belgique
Contact e-mail: pierre.job@ichec.be	Contact e-mail: jeanyves.gantois@ichec.be