How applicationism impact modelling in a Belgian school of economy and the viability of an alternative epistemology

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This paper deals with the following issue. Showing students how mathematics can be applied to economy in an applicationist way seems to make them unable to grasp the relevance of using mathematics to study economy. In other words, we expose an example of how a peculiar epistemological standpoint about the relationship between mathematics and economy, namely that of subordinating economy as an application of mathematics, may impact students' views about the interplay between mathematics and economy. We give an example to illustrate this issue and then conclude by giving hints as to an alternative way to articulate mathematics and economy, namely using economy as a semiotic foundation of mathematics.

Keywords: teachers' practices, applicationism, IBME, modelling, economy, ATD.

INTRODUCTION

According to Florensa and al. (2019), Inquiry based Mathematics Education (IBME), has spread, across the world, over the last two decades, being promoted by governments and international organizations through different means. Among these, curricula reforms and specific programs: for instance, PRIMAS and Fibonacci in Europe. Belgium is no exception to this trend and has, over the years, seen its curricula reformed, at the primary and secondary education level, to take into account different aspects of mathematics embodied in IBME, such as mathematical modelling and its relationship to the world. Despite this shared trend, implementations of IBME may put on various clothes, even at the research level, as noted by Artigue and Blomhoj (2013). This variety of approaches justifies that we take a closer look at the Belgian context and thus contribute to the study of IBME. In this paper we will focus on the tertiary level of education and more precisely on the setting of a Belgian high-school[1] in economy, business and management (school of economy in short). We will rely on the following research questions as guidelines. What form does IBME take in a school of economy? In particular, what kind of relationship does a school of economy have with mathematical modelling and economy? What are the factors that might impede or on the contrary facilitate the diffusion of an IBME approach in a school of economy? The aim of this paper is not to answer those questions in a definitive manner but more modestly to provide the following elements of a response. In section 3, we show how difficult it can be for mathematics teachers to engage in genuine modelling activities relevant to economy and how their relationship to mathematics tends to turn modelling into a form of applicationism (Barquero et al., 2013). In section 4, on the other hand, we explain how economy itself might provide a platform to implement a form of

[1] In Belgium, a high-school is university level institution.

IBME, namely a study and research path (Chevallard, 2015). Before getting to these sections we present our theoretical framework as well as relevant literature that puts our research questions in perspective.

THEORETICAL FRAMEWORK

Our theoretical framework is the Anthropological Theory of the Didactic (ATD) developed by Chevallard (1992). The use of the scale of levels of didactic co-determination (Chevallard, 2002) has proved to be a fruitful formalism to study didactic phenomena through the lenses of constraints acting upon institutions and knowledge. In relationship to IBME, Chevallard (2015) puts forward a high-level constraint to the diffusion of IBME. In short, IBME can be considered as an expression of a certain didactic paradigm, that of *questioning the world* (QW). And this rather novel QW paradigm conflicts with a much older one, that of *visiting works* (VW) which is still more spread and rooted in, at least, our western culture. The VW paradigm amounts to approach knowledge as a "monument that stands on its own, that students are expected to admire and enjoy, even when they know next to nothing about its *raisons d'être*" (Chevallard 2015, p.175). On the other hand, the QW paradigm (Chevallard 2015, pp.177-180) starts with a *generating question* tackled by a set of students and a set of guides of the study that together form a *didactic system* whose aim is to generate a final answer to the generating question. This final answer is the culminating point of *moments of study* of available information and *moments of research* that generate intermediate questions and answers. This particular relationship to knowledge delineates what Chevallard calls a *research and study path* (SRP). From these descriptions we can see that these two paradigms are mostly mutually exclusive. Barquero and al. (2013) go further and deal with constraints specific to university level in natural sciences in Spain. Among other things, they show how a certain dominant epistemology called applicationism, which considers that "mathematics has to be introduced by itself, having its own rationale, before being applied to extra-mathematical situations" (Ibid, p.317), tends to greatly restrict how mathematical modelling is understood (Ibid, p.317): "Under its influence [applicationism], modelling activity is understood and identified as a mere application of previously constructed mathematical knowledge or, in the extreme, as a simple exemplification of mathematical tools in some extra-mathematical contexts artificially built in advance to fit these tools". Do similar restrictions apply in our context of a Belgian school of economy? If yes, to what extent?

APPLICATIONISM IN HIGH-SCHOOL: THE BUDGET LINE EXAMPLE

To answer those questions will shall look into the first mathematical course students have to attend in our school, the way it was given between 2012 and 2017. Although it underwent many changes over the years, in terms of teachers, numbers of students, number of dedicated hours and even content, one topic remained the same. It is that of first-degree equations and lines (lines in short). We will focus on this topic as an invariant of the course able to inform us about the relationship to applicationism and modelling *over a substantial period of time*. The content of the chapter devoted to lines summed up to exposing the mapping between equalities of the form $ax + by = c$ and lines in a Cartesian plane. It was structured using the following mould. First the theory was recalled and then some routine exercises were provided for students: an equality was given and they had to draw the corresponding line in a plane and vice versa.

In 2013, an attempt was made for a brief period of time (a few sessions) were teachers decided to reverse the traditional order between theory and exercises. They did so because they had to face the fact that students were losing interest in the theoretical part of the course, in part because students were familiar with lines since their 3rd year in secondary school (14-15 years old) and were recalled for 4 years in a row the same topic. This departure from the norm turned out to be a failure. Students were not able to solve the exercises (no more or no less than when they were first given the theory) but this time they moreover complained they didn't have any theory to rely on and apply. Teachers felt guilty and entrapped because the only perceived way to keep the course going was for them to recall the theory anyway. This episode reinforced most of the teachers in their believe that "theory then exercises" was the only way to go. This shows a first linkage to applicationism. For most teachers in our school, the only possible way to teach mathematics is to expose the abstract theory and then apply it to some routine exercises, because their students don't have the required mathematical autonomy to learn outside the framework of a well-established theory.

A second linkage to applicationism is the following. It was no conceivable for teachers to leave aside the chapter on lines because lines were to be used in a subsequent chapter were elementary linear programming problems were solved using a geometrical presentation based on lines. Thus, students had to master the theory of lines before they could possibly encounter linear programming problems in a fruitful way. In other words, these teachers did not conceive that it could have been possible to teach mathematics in a manner that does not mimic its deductive architecture. The meaning of concepts is not driven by problems but rather by their logical embedding which is considered as the quintessential level of rationality of mathematics.

This reduction of teaching mathematics to architectural aspects had deep consequences on how modelling was treated by teachers as well. Being in a school of economy, teachers felt important to deliver a course that would be closely linked to economy. This desire to relate the two subjects was implemented by inserting "economic applications" into the course. They felt that doing so they contributed to introduce students to mathematical modelling, thus showing the relevance and importance of mathematics to economy. In the case of lines, the mathematical application considered was that of a budget line.

It was presented to students in the following manner. First a numeric example was given to them: "If two goods can be bought at respective prices of 2 and 5 euros, and if we have to buy a quantity x of the first good and a quantity y of the second in order to spend exactly 100 euros, then the equality $2x + 5y = 100$ must hold true. Thus, 100 euros outlays can be represented by a line in the plane of all possible outlays. This line is called a budget line".

Following this numeric example, a "general" version with letters was given in exactly the same way leading to $p_1x + p_2y = B$ with p_i standing for the unit price of good i, x and y being the respective quantity and B the budget at disposal. $p_1x + p_2y = B$ could also be represented by a line according to the theory that had been recalled earlier in the course. The budget line application was then over and students were given exercises like "Draw the budget line that represents the outlays related to two goods that can be bought at respective prices of 7 and 14 euros with a given budget of 1400 euros".

The way budget lines were introduced is very informative as to the impact of applicationism on modelling activities. Let us turn to students' relationship to budget lines to better understand the underlying mechanisms.

If we start with the exercises on budget lines like the one mentioned above, students behaved in a similar fashion as with exercises on abstract lines as soon as they had understood that they had to take $p_1 = 7$ and $p_2 = 14$ and $B = 1400$ to solve the exercise (and they did because teachers told them when they were stuck). They were then able to comply to the teachers' expectations by relying on the didactic contract (Brousseau, 2002) when they were not able to understand on their own what was required from them to solve the exercise.

But when we interacted with students, asking them what was the point of these budget lines according to them, many of them told us that they didn't feel at ease. For them, it was like they couldn't grasp the difference between budget lines and abstract lines, only in one case they had to use some economy related terminology (budget line, goods, …) but not in the other case. They could not figure out was budget lines were really useful for but they didn't bother too much with these concerns, because they could do the exercises and convince themselves that it is natural in a school of economy to have some economic terminology percolate through mathematical courses.

With this example, we can measure the gap that sets up between students and teachers, gap hidden under the appearance of the ordinary functioning of a regular course. Essentially, for students, budget lines don't make much sense and are definitely not the end product of a modelling activity as would be the case in a genuine study and research path. Indeed, there is no economic problem to which budget lines are an appropriate answer, the way the course was given, despite the fact that such problems seem pretty obvious and a priori within the reach of students. The following questions for instance might contribute to design an SRP. Is this possible to buy that amount of these two goods given that budget? What budget would be required to be able to buy that amount of these two goods? In other words, presenting budget lines the way they were lacks some fundamental character in the sense of Brousseau (2002). How did we get to that point?

The prevalence and naturality of the VW epistemology among teachers, of which applicationism is an offspring, tends to hide in the back *open* questions and problems in favour of a theory whose power to solve *closed* questions and problems (questions and problems designed to be solved by that theory) will justify its prominence. As a consequence, the idea to develop mathematics from the need that arise to solve an initially open problem is mostly absent. Instead teachers tend to reduce and focus teaching on the design of the best outfitting in which a theory should be dressed to minimize students' reluctance. The more energy they put in the design of such outfitting, the more they are unable to get in touch with their students' epistemic concerns because, from their teachers' point of view they made everything possible for students to understand the theory. This outfitting can take on the form mentioned above of using concrete numeric examples before using letters for the theory. This way, teachers feel they are really engaging in mathematical modelling and making it accessible to students, whereas students don't understand what is the point of budget lines besides learning some economic related terminology. It shows the mechanism by which the applicationist point of view deprives itself from the ability to design an economic problem where a mathematical model would be relevant to solve a genuine economic problem.

Chevallard (2015) relates the VW epistemology's long prevalence to the "social structure of formerly undemocratic countries" (p. 175) among other aspects. In the case of the interplay between economy and mathematics we may invoke another reason. The famous title of Wigner (1960) "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" is symptomatic of a train of thought that dates back to at least Galileo stating that mathematics is the natural language of nature and thus by extension the signature of any approach that would qualify as scientific. Although complex, the penetration of mathematics within economy can be partly related to this trend. Economy had its proponents to turn it into a hard science and not a "mere" social and human science and thus mathematize it: "« Avec sa théorie de la valeur, Debreu développe une approche résolument axiomatique dont le critère exclusif est la cohérence logique et non le rapport à la réalité » (de Vroey, 2002). In this context, being able to subordinate economy as an application of formal mathematics may be considered an achievement that equals the historic refoundation of physics and geometry based on "pure" mathematics. Thus, the way mathematics and economy interact at a pedagogic level is tainted by the means through which economy established itself as a hard science. From the perspective of economy that wants to establish itself as a hard science, the ability to be subordinated to mathematics is considered as a mark scientificity and this translates to mathematical courses given to economy students. These courses tend to be display economic application the way it is illustrated above with the budget line e.g. "pure" mathematics are developed with no connexion with economy and then "applied" to economy.

USING ECONOMY AS A POSSIBLE SEMIOTIC PLATFORM TO DEVELOP A STUDY AND RESEARCH PATH

In this section, we provide empirical data showing that economy itself might be used to develop a modelling activity where the meaning of mathematical concepts relies on the semantic of economic ground and thus showing a possible way out of strict applicationism. During the period 2012-2017, part of a chapter dealing with linear programming problems was devoted to the teach students that inequalities of the form $ax + by \leq c$ could be represented by half-planes and vice versa. This result was then used to give a geometric representation of linear programming problems that would allow to solve them geometrically. The argument used by the teachers was purely mathematical with no reference to economy and relied on the decomposition $\{(x, y)|ax + by \leq c\} = \bigcup_{k \leq c} \{(x, y)|ax + by = k\}$. The idea was to show students that a half-plane can be seen as a stacking of lines and thus reduce the study of the geometric representation of inequalities to that of lines (which had been recalled in a previous chapter). It turned out that students agreed on the geometric idea of a half-plane being a "stacking of lines" but irrespective of the above decomposition. They didn't not understand why the decomposition was used to assert that a half-plane is a "stacking of lines" as for them it was self-evident. As consequence they didn't understand either how to use the decomposition to draw the half-plane representing a given inequality. Teachers themselves had much trouble understanding what students couldn't understand in their argument. All in all, teachers felt pushed to leave aside the theory of inequalities and fall back on teaching what algorithm to apply to draw half-planes from inequalities. In 2017-2018, we had the opportunity to depart from the way the course was taught during the period 2012-2017 and were able to experiment on a small scale a different approach to inequalities and half-plane. This experiment is rather modest but nevertheless meaningful in our context because being in charge of hundreds of students does not allow much room for ideas that would be considered as "failures", by the institution. So, we had to make adjustments in the course very cautiously, in a step by step fashion, that would make changes not appear as dramatic modifications. The economic context used is the following. We have a budget of 400€ that allows to buy two types of tea. The first type T_1costs 5€/100g and the other one T_2 costs 4€/100g. The experiment can be divided in steps. We will fly over the first steps as we do not have enough space to details all of the experiment and focus on the steps directly related to the mapping between inequalities and half-planes. Step1. Through a set of questions like the following, students are led to calculate numerical expenditures: can you give expenditures spending the entire budget, can you give give expenditures spending more than the budget, given two expenditures which one is more costly, etc. This step allows students to feel at ease in the chosen context and relate the required calculation to an economic context that makes sense to them. Step 2. It also prepares them for the next step which seeks to make them move from the numerical register to algebra with the use of questions asking them to reflect upon expenditures like the following: what calculation do you have to make to determine if an expenditure spends the entire budget? This step leads to ostensive objects like $5q_1 + 4q_2 = 400$ where (q_1, q_2) denotes a certain expenditure. Students are then asked to give a geometrical representation of $5q_1 + 4q_2 = 400$ in a plane so as to be able to visualize all the expenditures spending the entire budget. Given their previous acquaintance with the topic, students are well aware that it gives rise to a line, even if they may have trouble drawing that line. The next step is what interests us most for our purpose. Step 3. Based on the following economic we lead students towards inequalities. In one company, the workers which to spend all the allotted budget for tea because at the end of the year, the amount that hasn't been spent returns to the company. In another company, budget is handled in a different way. If the budget is not spent entirely the remaining is left to the workers. The only condition imposed by the company is to not spend more than the initial budget. This second context leads to the expression $5q_1 + 4q_2 \leq 400$ which models whether or not an expenditure (q_1, q_2) will exceed the allotted budget. Student are then asked to also give a geometric representation of $5q_1 + 4q_2 \leq 400$ and contrast it to the previous context of $5q_1 + 4q_2 = 400$. At this point students are not accustomed to expressions like $5q_1 + 4q_2 \leq 400$ even though they have already met inequalities in secondary school. What is interesting for our purpose is that some students were able to give a geometric meaning to $5q_1 + 4q_2 \leq 400$ based on an economic reasoning. The trail of their reasoning is the following.

- Some students note, for instance, that $q_1 = 40$ and $q_2 = 50$ exhausts the 400 € budget.
- It thus means that any increase of q_1 or q_2 will exceed the budget. And any decrease will no exhaust the budget.
- The geometric consequence is that starting from a point on the line representing $5q_1 + 4q_2 = 400$ like $q_1 = 40$ and $q_2 = 50$, increasing wether q_1 or q_2 or both at the same time, will give birth to a point in the plane (q_1,q_2) that will be located "above" the line. A similar conclusion can be drawn while decreasing those quantities. Such points will all be located "below" the line.
- From these considerations, students are able to give an economic meaning to the interplay between algebra and economy. Expenditures spending less than the budget verify $5q_1 + 4q_2 < 400$ and are geometrically located "below" the line represented by $5q_1 + 4q_2 = 400$. Points exceeding the budget verify $5q_1 + 4q_2 > 400$ and are geometrically located "above" that same line.
- Thus geometrically, $5q_1 + 4q_2 \leq 400$ can be divided into points on the line $5q_1 + 4q_2 = 400$ and points "below" it $5q_1 + 4q_2 < 400$.

The way these students reasoned about the geometric meaning of $5q_1 + 4q_2 \leq 400$ is remarkable in our institutional context for several reasons. First it shows that it is possible for students to take responsibility of a fragment of interplay between algebra and geometry. To the best of our knowledge, it never happened in our institution in the framework of the course we are studying in this paper: all theoretical aspects have always been taken in charge by teachers. It means that students can be made much more responsible than thought and have the ability to contribute to the development of a theory. Second it shows that applicationism is not the only “possible” way to teach mathematics in a school of economy. Third, the mapping between algebra and geometry performed by students relies on economy i.e. economy is used to give a geometric meaning to algebra. We would like to stress this aspect because it shows not only that something different than applicationism is feasible but that the subordination of economy to algebra and geometry is not inescapable. Economy can be envisioned as a stepping stone on which mathematics can be build whose objects’ semiotic relies on economy like for instance inequalities.

CONCLUSION

We showed how much applicationism is rooted in our school of economy. It means that it underlies the whole course that has been studied but more than this, that teachers feel difficult to teach another way because attempts to modify the “theory then exercise” model turned out to be failures. One reason for this failure worth exploring in another paper would be the idea that teachers lack other levels of rationality than the deductive one which deprives them from envisioning the teaching of mathematics according to other organizing principles. As a consequence of their failed attempts, it seems to comfort them with the idea that teaching mathematics mostly consists in thinking about which outfitting should be used to wrap the theory they want to teach in a way that minimizes frictions with students. From this perspective, the “theory then exercise” model appears to be a generic outfitting that allows to use exercises as a mean to discharge students from taking responsibility and making sense of the theory: when students are successful at exercises it is considered as a mark of understanding.

We also showed of much applicationism impacts modelling. It reduces mathematical modelling of economy to applying “pure” mathematics to economy. This has a potential impact on students’ perception of economic applications. The case of budget lines suggests that presenting them as a mere application of mathematics deprive students for the ability to consider this application as meaningful. This leads to a vicious circle. Teachers feel that economic applications give more credit and substance to the usefulness of their course by tightly interacting with economy, when in fact, the very way it is presented to students has the reverse effect on them as it deprives them from the possibility to understand which relevant economic problem has been tackled.

These results around applicationism and modelling are in line with those found in Barquero and al. (2005). It would be interesting to study the extent to which such phenomena apply in other institutions in Belgium and around the world but also within our institution in other courses which we so far had no access to.

Lastly we showed that it is possible, even if it was experimented on a small scale, to develop some mathematics starting from economy where economic can act as a milieu which students can interact with to construct a semiosis that connects first order inequalities to half-planes thereby showing the possibility to deconstruct the applicationist paradigm and opening to a tighter integration between mathematics and economy.

The ability to make mathematics rely on economy is, as noted before, an important result, at least in the context of our institution. It nevertheless raises a question that might at first seem to downplay the relevance of this result on which we will end this paper. To what extent the use of economy as the semiotic foundation of mathematics might contribute to create epistemological obstacles? Indeed, if we imagine a course entirely built on economy, it might lead students to not be able to grasp the meaning of mathematical concepts in any other way than being rooted in economy. We think for instance of mathematical structures that emerge from needs internal to mathematics.

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