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Dealing with the Heterogeneity of Interpersonal Relationships in the Middle Ages. A Multi-Layer Network Approach

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Investigating the case of the Investiture Struggle in the diocese of Cambrai–Arras (c. 1100), this article aims at exploring some crucial issues for historians using social network analysis in the study of heterogeneous relationships. The study proceeds along three lines of enquiry. First, by establishing a hierarchy in the different types of relationships mentioned in the sources, it determines which of them are the most important to model and understand the structure of the network. Second, it demonstrates it is unnecessary to consider co-witnessing relationships (i.e. to be witnesses of a same charter) in the modelling of networks. Indeed, co-witnessing relationships do not help to improve our understanding of the structure of the parties at stake in a conflict. Finally, this paper deals with the importance of rank order in the witness lists. It demonstrates that, in the case of Cambrai, rank order does not have an influence on the global structure of the network. In other words, all individuals in the same witness list play a similar role in the network in terms of party structuring.

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I. Introduction

§1 For a few years now, social network analysis has become a relatively common tool for medievalists working on political history. Following in the footsteps of John Padgett and Christopher K. Ansell on the Medici (Padgett and Ansell 1993), many historians have devoted articles and books to networks of power in the Middle Ages. Exploring new explanatory models, these works, such as the articles of Isabelle Rosé (2011; 2018) or these of Robert Gramsch-Stehfest (2013; 2018) and Matthew H. Hammond (2017), have underlined the critical and methodological issues that occur in the use of medieval documentation in social network analysis.

§2 Some difficulties among these are related to the heterogeneity of the set of social and political relations being modelled. Very often, the medieval sources used to construct the analyzed graphs make us aware of a whole range of links between the people mentioned in them, whose natures are multiple and diverse. This observation is particularly true when the basis of the medievalist's work is a diplomatic corpus consisting in charters describing various legal actions. In this case, the heterogeneity of relations appears at three levels. First, the relationships attested in charters of different legal types are different in nature. For example, the protagonists of a donation charter do not have the same relationship as those of a conflict resolution document. Second, within the same charter, different roles give rise to different relationships. For instance, one could expect that the author-beneficiary link is not identical to the author-witness link. Third, even for equal roles, it is relevant to ask whether the relationships between individuals are quite the same. Does a charter describing the donation of a good to a group of beneficiaries necessarily attest to links between these beneficiaries and the donor that are all equal in intensity?

§3 In this article, we study some methodological issues related to these questions with the help of quantitative methods. To do so, we explore the case of a complex, but "classical", political conflict occurring in Cambrai towards 1100 in the context of the Gregorian reform. We have chosen this case study because the events have already been studied through a qualitative approach by one of us (Ruffini-Ronzani 2014). The global structure of the conflict, its main players and their distribution between the different parties at stakes are well known. Consequently, the "traditional" analysis allows us to verify whether the quantitative approach gives coherent results or not.

§4 Our methodology is based on a multi-layer network approach. Its main idea is to decompose the considered graph into subgraphs corresponding to different relation types (the "layers"). We can thus consider those layers separately, apply transformations to them, and aggregate them into a new single-layer network. The resulting graph is finally compared to the original undecomposed graph with the help of a procedure that automatically reconstructs the parties of the conflict.

§5 The application of this framework to the Cambresian network allows us to address three methodological issues. First, we ask whether all categories of edges in the graph (i.e., all categories of relationships) appearing in the charters contribute equally to our understanding of the political conflict under study. The interest of obtaining a hierarchy of types of relationships in terms of efficiency in the modelling of a historical phenomenon is twofold. On the one hand, it helps to guide the historian who is building a network on the basis of a diplomatic corpus, indicating which links he should pay particular attention to, etc. On the other hand, such a ranking of socio-political links in order of importance tells us something about the party structure of the Cambrai conflict, and more generally about the nature of interpersonal relations in the Middle Ages.

§6 Second, we look at a particular type of relationship that appears only indirectly in the diplomatic texts: co-witnessing (which we will also call co-subscription). This relationship binds together the witnesses of a same charter, those individuals who appear on the lists copied at the end of the charters and whose function was to validate the legal action inscribed in the act (Guyotjeannin, Pycke, and Tock 2006, 89). Whether these links are important and to what degree they play a role in the structuring of socio-political relations in the period are important questions for historians who build networks from the documentation of the early and high Middle Ages. This issue is not anecdotal, because taking into account this category of links is often difficult to implement in practice. Indeed, these relationships are often very numerous: witness lists can be long (up to 41 witnesses in our corpus, with an average of 14.7), and the number of resulting edges is of the order of the square of their length (e.g. 820 edges for 41 witnesses). Including them in an analysis can therefore significantly deteriorate its results, whether they are visualizations (unreadable plots) or statistical calculations (bias introduced by their overwhelming number compared to relationships in other categories: in the case of our network, it multiplies the number of edges by ten). It is thus important to know if considering them is worth the effort.

§7 Third, we are interested in the relations that the witnesses have with the authors of the charters, and more precisely in the order in which they are mentioned in the list given at the end of the act. According to historiography, this order is not due to chance, and carries a real meaning for medieval men (Genicot 1972, 41–43; Keefe 1997). It is easy to imagine that the first places on witness lists were reserved for individuals of high social rank, and whose testimony is therefore of great value. However, some questions remain open in this regard. Does this hierarchy of honours have an impact on the importance of the corresponding subscription relationships? In particular, is the link between the author of a charter and the first witnesses stronger than the one between him and the last witnesses? Can these last witnesses simply be omitted in a

network analysis? The framework implemented in this article allows us to consider answers to these questions.

§8 Our argument will follow three steps. In the first part of the paper, we will present briefly the political context in which the Investiture Contest in Cambrai occurred. We will also describe the corpora we have used, focusing primarily on the eleventh- and twelfth-century episcopal charters of Cambrai. Then, we will turn to the methodology we have developed to address the difficulties historians have experienced while dealing with medieval relationships heterogeneity. Finally, we will present our results, by focusing first on the automatic labelling of the parties, then on the answers we propose to the three questions asked here above.

II. Cambrai during the Investiture Crisis: Context and sources

§9 In the last few years, French historiography has underlined the role of the Gregorian reform in the transformation of society in the high Middle Ages. According to Florian Mazel, the Gregorian reform was a "global revolution" by which not only the political structures but also the mentalities were transformed (Mazel 2010, 233–298). Whether one agrees or disagrees with this statement, one must admit that this new paradigm of French historiography has some explanatory value. It is particularly true in the case of Cambrai. The history of the diocese of Cambrai is complex. Since the seventh century, the bishops of Cambrai ruled over the dioceses of Cambrai and Arras, which constituted a double diocese located in the "French" ecclesiastical province of Rheims. Since the beginning of the eleventh century, the bishops were also counts of the county of Cambrai, a small principality settled in the Empire, at the borders with the French Kingdom and the county of Flanders. In the context of the "Imperial Church System", the bishops largely benefited from the support of the Emperors (Ruffini-Ronzani 2016; 2019). However, the situation changed at the end of the eleventh century, as the canons of Arras sought the independence of their diocese (Cauchie 1890–1891; Delmaire 1994; Kéry 1994; Resnick 1997; Van Mingroot 1991). The pope and the count of Flanders gave their support to their struggle, because it served their political interests. Conversely, the Emperor and the Church of Cambrai were opposed to the split of the diocese of Cambrai–Arras, because it constituted a German beachhead in France. At the same time, two candidates were elected bishops in Cambrai. On the one hand, Walcher of Oisy was a former archdeacon in Brabant and the candidate supported by Emperor Henry IV and then his son Henry V. On the other hand, Manasses of Eu-Soissons was a former canon of Rheims. He was supported by the pope, the archbishop of Rheims and the count of Flanders. Each of the candidates had some supporters among the aristocracy of the county. These events resulted in a civil war. From 1092 until 1107, Walcher and his

allies waged war against their opponents in the context of the Investiture controversy. Walcher's main opponents were the pope, the count of Flanders, Bishop Lambert of Arras – the newly elected bishop of Arras –, and Bishop Manasses, who was replaced in 1105 by Bishop Odo, the former abbot of St Martin Abbey in Tournai. The conflict was closed by the victory of the "Gregorians" and the count Robert II of Flanders against the Emperor's supporters. The treaty of Aachen put an end to the struggle in 1107. However, the political tensions only calmed down in 1113, with the death of Bishop Odo. The main losers of the conflicts were Emperor Henry V and, moreover, Walcher of Oisy. The latter was forced to renounce to the bishopric of Cambrai and reintegrated by the pope in his former position of archdeacon of Brabant.

§10 The conflict that devastated the county of Cambrai at the turn of the eleventh and twelfth centuries is known through three types of sources: a) The numerous charters issued by Bishops Walcher, Manasses and Odo of Cambrai (Van Mingroot 1995); b) The *Gesta Galcheri*, a polemical chronicle written in the entourage of Bishop Walcher between 1113 and the end of the 1120s (*Gesta Galcheri* 1883); c) The "register" of Bishop Lambert of Arras, a twelfth-century manuscript containing a lot of letters dealing with the independence of the bishopric of Arras (Giordanengo 2007). Each of these sources offers a different standpoint on the struggle. They are complementary, as each of them sheds a different light on the same reality (Ruffini-Ronzani forthcoming a). Our article will deal mainly with charters. Nevertheless, we will also use the other sources, as they help to understand how structured the parties in conflict were.

§11 In order to analyse the struggle, we have reconstructed the ego (or personal) networks of the three bishops claiming the episcopal see of Cambrai. Therefore, it was necessary to gather all the charters produced in their name (an example of which is shown in **Figure 1**). Episcopal charters were relatively numerous in Cambrai towards 1100 in comparison with other dioceses (Van Mingroot 1969). According to Van Mingroot (1995), we have still 11 charters for Walcher of Oisy (1093–1107), 22 for Manasses of Eu-Soissons (1093–1103) and 51 for Odo of Tournai (1103–1113), that is to say 84 charters in total (without taking into account false charters). Among them, 32 are original charters, 48 are copies, and 4 are fragments. These charters usually end with a witness list. The number of witnesses varies considerably from one list to another (from 2 to 41, as in a prestigious charter of 1095 for St Aubert Abbey in Cambrai). The average number of witnesses is 14.7 (23.7 for Walcher, 15.7 for Manasses, 11.9 for Odo). The witness lists constitute the part of the charters giving the most information about the personal networks of the bishops. However, their analysis may not be sufficient to understand the structuration of the episcopal entourages. Indeed, it seems also necessary to represent the links between the members of the bishops' entourage that

do not pass through the prelates to pinpoint the presence of clusters of solidarities in their entourages. This explains why we also include in our corpora the charters of the members of bishops' entourages (Rosé 2011).

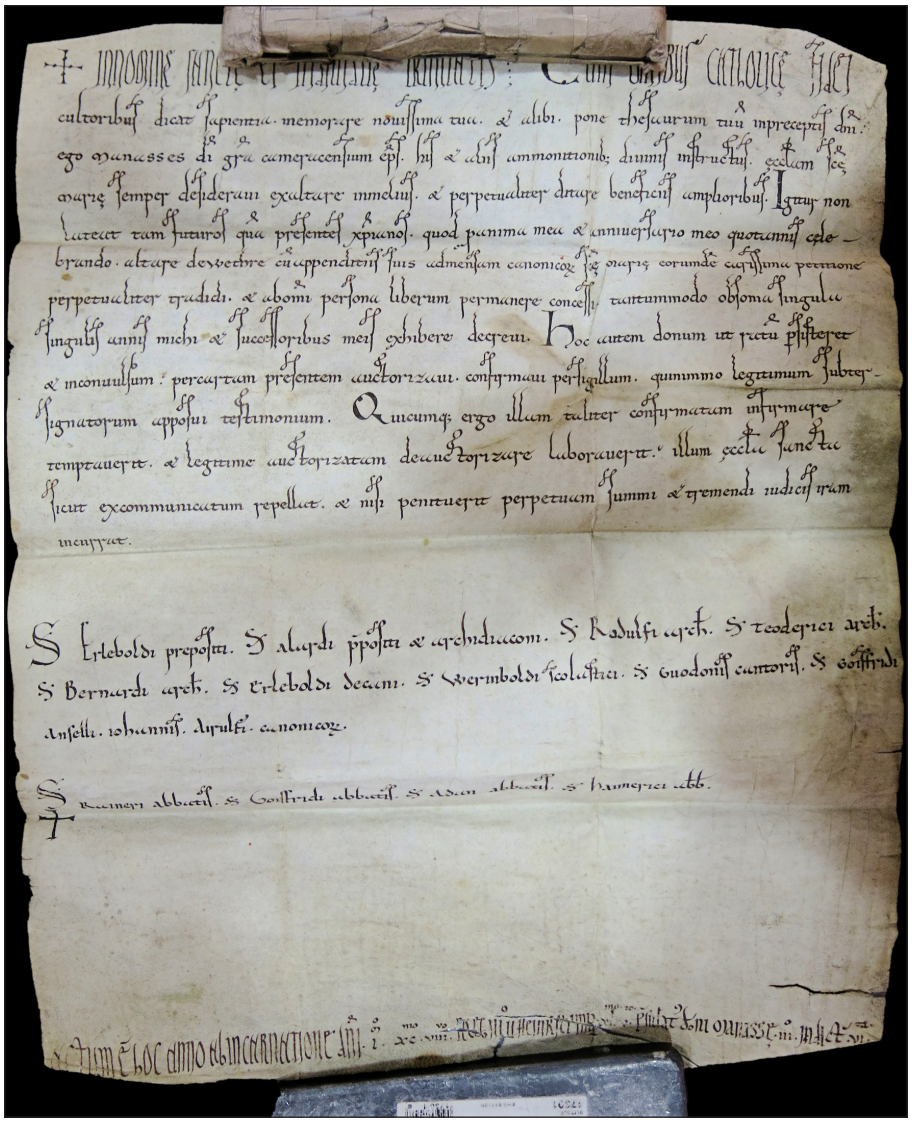

Figure 1: One of the charters under consideration. The witness list follows the capital S in the bottom half of the page (*Diplomata Belgica* 2015, DiBe ID 3545).

§12 We will only use the two other sources that are informative about the Investiture Contest in comparison with the charters, as explained at the end of the previous section. One of the best sources of information about the conflict is the *Gesta Galcheri*, a historiographical text written in the entourage of Bishop Walcher between

1113 and the middle of the 1120s (*Gesta Galcheri* 1883; Ruffini-Ronzani forthcoming b). In other words, this text was written at a time when the war was over but remained in all memories. This explains why the *Gesta* are so accurate about the struggle. The accuracy of the *Gesta Galcheri* is also explained by the profile of their anonymous author, as there is no doubt he was a fierce supporter of Walcher. The text recounts the struggle in Cambrai from the point of view of Walcher's supporters. In this long text, we have therefore the version of one of the main losers of the conflict, which is quite uncommon in medieval sources. The *Gesta* mainly focus on the powerful players of the war, among which Bishop Walcher is obviously the key protagonist. For their author, the conflict in Arras and Cambrai was less a civil war than an international conflict. For instance, he barely mentions the role of the lower aristocracy and the interventions of the local clergymen. In addition, the conflict is represented in a very caricatured way. The role of each player in the conflict is described explicitly. Walcher's enemies are often described in polemical terms, such as "heretic" or "apostate", for instance. According to the source, the Emperors, the duke of Lower Lotharingia and the bishop of Liège were the only supporters of Bishop Walcher, who would have been quite isolated on the political field.

§13 The "Register" of Bishop Lambert of Arras gives a complementary image of the conflict. The term "Register" is somewhat ambiguous. In her recent edition, Claire Giordanengo (2007) refers to it as a collection of letters, charters and narratives produced during the episcopacy of Lambert of Arras and copied in a seventeenth-century manuscript. In other words, the "Register" in the sense of Giordanengo is first and foremost a production of an early modern scribe. What we will call "Register" here is something different. In fact, approximately forty of these texts (mainly letters) were copied at the very beginning of Lambert's episcopate in a manuscript that also contains liturgical texts, a list of the popes, a list of the archbishops of Rheims and a list of the bishops of Cambrai and Arras since the beginning of the Christian Era (Boulogne-sur-Mer, Bibliothèque municipale, ms. 84). Steven Vanderputten (2014, 181) designates this book as the "manuscript fondateur" of the diocese of Arras, as it "lays out liturgical, juridical, historical and ideological arguments in favour of the independent bishopric of Arras". This source is particularly interesting as it gives another point of view on the struggle, which is here viewed in the perspective of the canons of Arras. The many letters copied in the "Register" reveal the names of the allies of Bishop Lambert and the canon of Arras in the Investiture Contest. The manuscript highlights the role of the papacy and the archbishops of Rheims. Bishop Lambert was frequently in contact with these allies for political affairs. The letters also indicate what were the main difficulties Bishop Lambert and the canons of Arras faced on at the turn of the eleventh and twelfth

centuries and who were their main rivals. Bishop Walcher and some other Cambresian players were among them. As is true of the *Gesta Galcheri*, it focuses mainly on the most prominent players of the conflict. **Figure 2** gives a visualization of the network we built by combining these three types of sources (more details about it are given in Section 3.1).

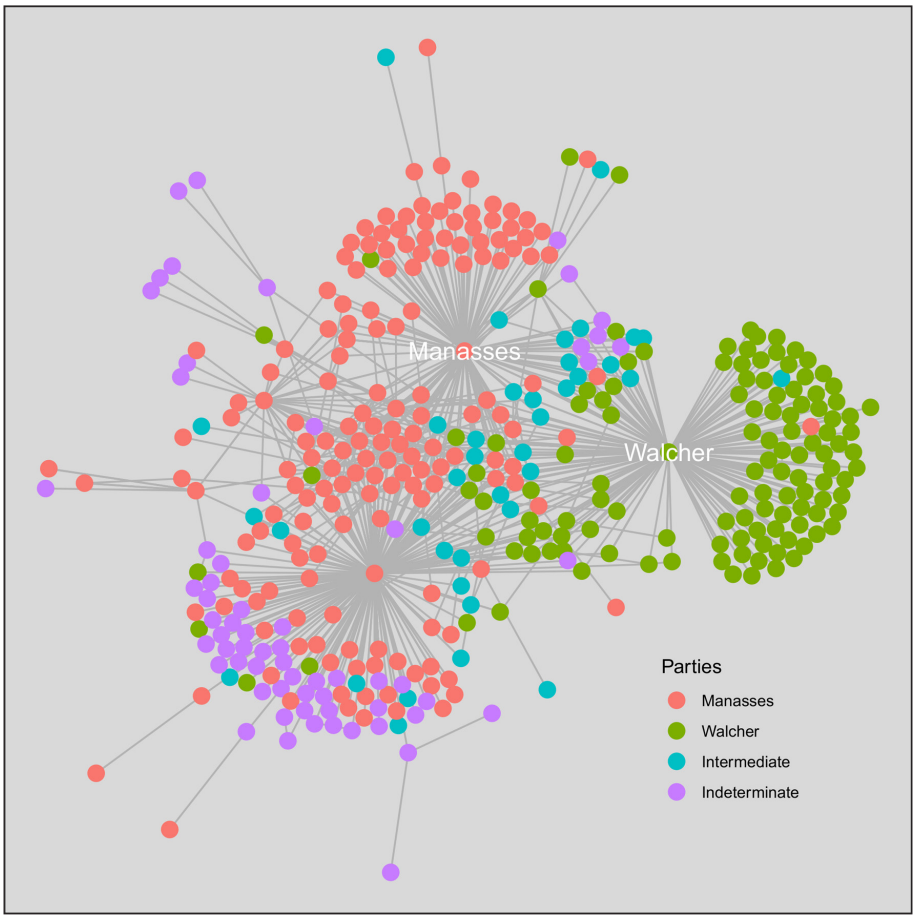

Figure 2: Network we obtain from the historian's expertise.

§14 The vertices and edges of the graph come from the information extracted from the charters only. The colors of the vertices represent the parties to which the corresponding individuals belong. These attributions were made manually. Four parties were defined, by analyzing the way the actors of the network are portrayed in the charters, the *Gesta Galcheri* and the Register of Lambert of Arras. The manual reconstruction of the parties was facilitated by using the prosopographical approach developed by Van Mingroot (1991) and thanks to previous works of the authors of this

paper. The first two parties correspond to the two opposing groups: Walcher's party and Manasses' party. A third party "Intermediate" includes people who do not lean to one side or the other. Finally, the vertices for which the attribution is too uncertain are classified in a fourth party "Indeterminate".

III. Methodology

§15 To answer the questions posed in the first section of this article, we have set up a methodology consisting of four blocks. The first one is a procedure for the automatic reconstruction of the parties in the conflict on the basis of a network. This procedure can be used on the network made up of the relations referred to in the diplomatic sources presented above. The second block consists in modifying this basic network by increasing the importance of some of its edges: to do this, we embrace the multi-layer network approach. Third, we compute for each network a metric that quantifies the quality of the parties in conflict that this network allows to reconstruct. Finally, we define a way to account for a hierarchy between certain relationships in the network. The comparisons that we obtain during these steps bring elements of answer to the questions asked.

§16 Before going into those details, we describe the network under consideration in the first subsection. The four other subsections are devoted to the description of the four methodological blocks. All our calculations have been made with the R software (R Core Team 2020) and the igraph library (Csardi and Nepusz 2006).

III.1. Graph modelling the Cambrai Conflict

§17 The 399 vertices of the network we consider in this paper correspond to the protagonists in the conflict presented in Section 2, and people who appear alongside them in diplomatic sources. The 10863 edges of this network, in turn, represent the relationships that these entities maintain in these sources.

§18 These edges have two attributes. The first one is a category, the nature of which represents the type of relationships the two entities maintain in the sources. **Table 1** gives an explanation of them and shows their distribution. Two of these categories are particularly high in number: Subscription and Co-subscription. Despite the bidirectional nature of some of these relationships, we have chosen to consider them all as unidirectional relationships. Considering oriented graphs poses a set of problems. Among them, as in this case, some vertices are isolated from the rest of the graph, and in particular from the pivot-vertices, i.e., there is no path connecting them to the rest of the graph. As we will see below, this is an issue for our methodology. All the graphs we manipulate are therefore non-oriented.

Relation category	Explanation	Number
Abbot	X is abbot of Y	57
Alliance	X enters into an alliance with Y	7
Consent	X consents to an action of Y	31
Cosubscription	X and Y both appear among subscribers of some charter	9457
Donation	X gives a property to Y (or confirms such a donation)	164
Kinship	X and Y are parents	17
Notice	X gives notice about an action of Y	29
Request	X requests from Y to take some action	54
Subscription	X appears among the subscribers on Y's charter	1047

Table 1: Distribution of relations categories within the Cambrai network.

§19 The second attribute of the considered edges is a weight, which is initially defined as the number of charters that attest to such a relationship between the two entities. As we will see, these weights will be modified in order to perform analyses. All the graphs we consider in this paper are therefore weighted.

III.2. Automatic reconstruction of the parties in conflict

§20 The first issue we deal with is the automatic reconstruction of the parties in conflict on the basis of a network, no matter how this network was built. Automatic reconstruction of historical political parties has already been undertaken in historiography (see for example Dahmen, Bazzan, and Gramsch-Stehfest 2017; Gramsch-Stehfest 2020). However, in the context of the Cambrai conflict, this task does not really consist in detecting communities in the sense generally understood by graph theory specialists. It is not a question of searching for "dense subgraphs" that are well separated from each other (Fortunato and Hric 2016, 5). We noticed that the main community detection algorithms (walktrap, spinglass, edge betweenness) were not giving good results. The resulting clusters almost always included some peripheral regions of the graph, which are not of particular interest in our two-party framework. It then became apparent that we needed to proceed in a different way, by forcing the computer to reconstruct plausible parties.

§21 Rather than detecting communities, we identify here the party with which each vertex is associated by using the connections between this vertex and some a priori defined pivot-vertices. It is more a task of graph-partitioning (or graph-coloring) than a task of community-detection. The pivot-vertices are those representing the two

candidates for the episcopal see, Walcher and Manasses. The min cut algorithm is a method for partitioning a graph into two subgraphs containing vertices fixed a priori (Stoer and Wagner 1997). However, the non-uniqueness of the partitions it generates, and the fact that it does not allow to obtain an Intermediate party, are obstacles to its use in our context. For this reason, we have implemented our own algorithm for reconstructing the parties, which is very simple: for a vertex *i*, we compare the distance (in a sense that will be specified below) between *i* and Walcher (which we will henceforth note *d(i, Walcher)*) and the distance between *i* and Manasses (*d(i, Manasses)*). If these two distances are too close to each other (i.e., if the absolute value of their difference is smaller than a threshold that will be discussed below), the vertex *i* is placed in the "Intermediate party". If the distance from *i* to Walcher is much smaller (in the sense of the threshold mentioned above) than the distance from *i* to Manasses, *i* is placed in the "Walcher party". If, on the contrary, the distance from *i* to Walcher is much greater than the distance from *i* to Manasses, *i* is placed in the "Manasses party". In short, each vertex is placed in the party of the pivot-vertex it is closest to, unless it is about equally distant from both. **Listing 1** shows this simple algorithm in pseudo code.

Listing 1: Graph partitioning algorithm.

```
for each vertex i,
    compute d(i, Walcher) ;
    compute d(i, Manasses) ;
    if |d(i, Walcher) − d(i, Manasses)| ≤ threshold, then party(i) = "Intermediate" ;
    else if d(i, Manasses) − d(i, Walcher) > threshold, then party(i) = "Walcher" ;
    else if d(i, Walcher) − d(i, Manasses) > threshold, then party(i) = "Manasses" ;
```

The threshold at which a difference in distance is considered to be significant has been set at the 10%-quantile of the series formed by all differences in distance:

$$\text{threshold} = 10\% - \text{quantile of } \{|d(\text{Walcher}, j) - d(\text{Manasses}, j)| \text{ over all vertices } j\}.$$

This is the choice that gave the best results (in a sense that will be specified below).

§22 The partitioning method described above is based on the concept of distance between two vertices of a non-oriented weighted graph, which we now need to define. The distance we use is very common (the use of another distance is discussed in Section 4.2). The weight of each edge is first transformed into a cost equal to the inverse of the weight (*cost = 1/weight*). Then, the standard Dijkstra shortest path algorithm is applied (Dijkstra 1959). The distance is equal to the cumulated cost of the shortest path.

III.3. The multi-layer approach

§23 To simultaneously process and analyze the interactions of the different categories of relationships in our graph, we embrace the multi-layer network approach. Rather than manipulating a classic network whose edges have an attribute that specifies its category, we consider our network to be the superposition of several sub-networks that we will henceforth call layers (this approach is well know in graph theory literature, see e.g. Kim and Lee 2015, and has already been used in contexts similar to ours, e.g. in Opitz, Born, and Natase 2018; Opitz 2020; Bornhofen and Düring 2020). Each of these layers is itself a graph, whose vertices are the same as those of the complete graph, and whose edges correspond to the edges of a particular category. We therefore decompose our complete network into nine layers, since, according to **Table 1**, its edges belong to nine different categories.

§24 **Figure 3** gives a simple example of such a decomposition (into four layers rather than nine, for the sake of clarity). This example network consists of three vertices (Odo, bishop of Cambrai; Ename Abbey; Gilbert, abbot of Ename) and seven edges, which belong to four categories (three times Donation, two times Subscription, once Request and once Abbot). It is therefore broken down into four layers, which correspond respectively to the edges of the Donation category, the Subscription category, the Request category and the Abbot category.

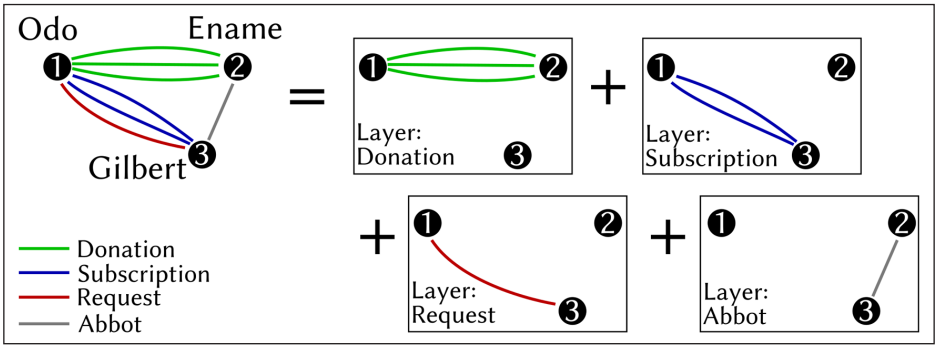

Figure 3: Decomposition of an example network into layers corresponding to its edges categories.

§25 In terms of adjacency matrices, the decomposition amounts to a simple sum of matrices: in the case of **Figure 3** example, we have

$$\begin{pmatrix} 0 & 3 & 3 \\ 3 & 0 & 1 \\ 3 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

§26 Note that, rather than decomposing the left-hand side matrix, this equation allows us to construct a single-layer network from a set of layers. If we know the adjacency matrices of all the layers (i.e. the right-hand side matrices), we can sum them to obtain the adjacency matrix of the multi-layer network resulting from the aggregation of these layers.

§27 It is also possible to apply operations other than the simple sum to aggregate the layers (as it has been done e.g. in Berlingerio, Coscia, and Giannotti 2011). Among these, we can think of any linear combination, in which each adjacent matrix is associated with a specific coefficient, i.e. a number by which the matrix is multiplied in the sum. When layers are aggregated using such a linear combination, the choice of the coefficients is an important matter. If the coefficient of one of the layers is greater than those of the other layers, the edges of the category associated with that layer will have a greater influence within the single-layer network resulting from the aggregation.

§28 For example, we could aggregate the adjacency matrices corresponding to the four layers of our example network by increasing the importance of the second category using a linear combination with coefficients 1, 3, 1 and 1:

$$\begin{pmatrix} 0 & 3 & 7 \\ 3 & 0 & 1 \\ 7 & 1 & 0 \end{pmatrix} = 1 \cdot \begin{pmatrix} 0 & 3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + 3 \cdot \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

§29 The adjacency matrix resulting from this linear combination (and the network it represents) gives pride of place to the edges of the second category (Subscription). By inflating the coefficient of this type of edges this way, we have made the relationship 3–1 pass in front of the relationship 1–2, and consequently we have modified the network in depth.

III.4. Assessing the relative importance of relations

§30 To evaluate the relative importance of the different categories of relationships in the Cambrai network, we consider the single-layer networks that result from an aggregation process similar to that of this last example. We use linear combinations in which all layers are associated with a weight of 1, except one, which is associated with a high coefficient (i.e., much greater than 1).

§31 It is then a question of comparing the single-layer networks generated this way, measuring to what extent they correctly reflect the political situation of the conflict. For each of them, we therefore apply the automatic reconstruction procedure presented in Section 3.1. We then compare the parties obtained this way with the

parties obtained by the manual partitioning described in **Figure 2** and derived from the historian's expertise. We treat this manual partitioning as a set of *a priori* known labels and place it in a context similar to that of supervised learning (see remarks on this subject in Section 4.2). The following ratio is used as a metric for this comparison:

$$\text{classification ratio} = \frac{\text{number of vertices classified in the correct party}}{\text{total number of classified vertices}}$$

Note that we exclude from this count the vertices that the manual partitioning puts in the "Indeterminate" class.

§32 For each category, we generate not one, but many single-layer networks, gradually increasing the coefficient associated with the category in question. We thus obtain a plot whose x-axis gives the coefficient that has been assigned to the category, and the y-axis gives the classification ratio.

III.5. Determining if the witnesses order is meaningful

§33 Working with the order in which the witnesses (subscribers) appear in a charter is a different task from that described in the previous section. It is no longer a matter of increasing the relative importance of one category of relationships relative to the others, but of changing the relative importance of relationships that all belong to the same category (the Subscription category).

§34 The idea is as follows. For each charter, we sort the Subscription relationships by the order in which the corresponding individuals appear on the document. We then assign a coefficient to these relationships according to the place they occupy: an item at the top of the list is given a large coefficient, while an item at the bottom of the list is given a small coefficient. The next step in determining the impact of the order of subscribers is to vary the scale of this range of coefficients, from a situation where the relationships are almost on an equal footing (all similar coefficients) to a situation where there is a large difference between the first relationships and the last (very different coefficients).

§35 More precisely, we use the following function: to the i-th relation of a set of n relations (which therefore corresponds to the witness placed in position i of a list of witnesses of length n), we attribute the weight

$$\text{coefficient}\,(i,\,\alpha) = 20\left(\frac{n-i+1}{n}\right)^{\alpha},$$

where α is a parameter whose positive value we vary, and the constant 20 has been chosen arbitrarily (its value does not matter, as long as it is much greater than 1). From this definition it follows that, for a fixed $\alpha > 0$, when i is close to 1, i.e. when the witness

is at the top of the list, the coefficient is large. In contrast, when *i* is close to *n*, i.e. when the witness is at the bottom of the list, the coefficient is small. Varying the parameter α changes the range of values over which the ordered list of witnesses is distributed.

§36 As an example, we consider the list of the seven witnesses of a 1108 charter notifying the settlement of a conflict (*Diplomata Belgica* 2015, DiBe ID 3850), a picture of which is given in **Figure 4**. The different sets of coefficients that are associated with these witnesses when α takes values between 0 and 10 can be seen in **Figure 5**. When α is set to 0, all witnesses are associated with the same coefficient. At $\alpha = 1$, the first witness, *Erleboldus decanus*, remains at the same coefficient, while the other six witnesses receive a smaller coefficient. This difference widens as the values of α become larger and larger. As can be seen in the plot, when this parameter is equal to 10, the last four witnesses receive a very small coefficient, close to 0. Only the first three witnesses then count.

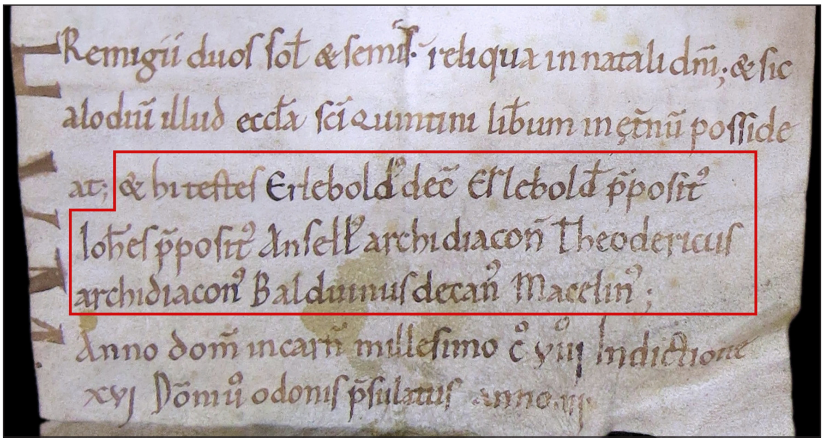

Figure 4: List of witnesses of a 1108 charter (Diplomata Belgica 2015, DiBe ID 3850).

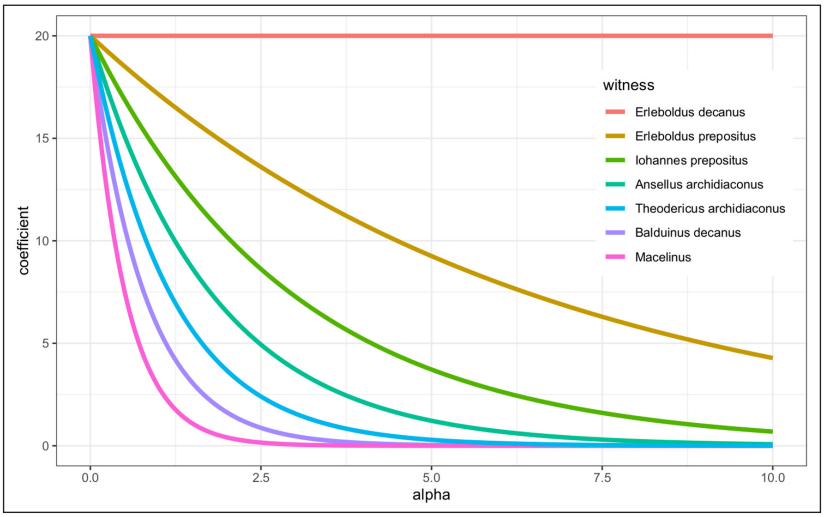

Figure 5: Evolution of the coefficients of ordered witnesses when varying values of parameter α.

§37 For each value of α, a single-layer network is generated by modifying the coefficient of the Subscription layer relations as described above. All these networks are then compared using the classification ratio defined in Section 3.3. This comparison makes it possible to estimate the importance of taking into account the order of the list of witnesses given in a charter.

IV. Results

IV.1. Quantitative findings

§38 In this section, let us first present and comment on the results of the automatic party reconstruction procedure. **Figure 6** shows the parties that are constructed this way (without considering cosubscription relations) and compares them with the manually defined parties. Each vertex is represented using two colors: the outer color gives the party that was manually attributed to it, while the inner color gives the automatically attributed party. **Table 2** gives the confusion matrix of this automatic procedure, allowing to analyze its performances.

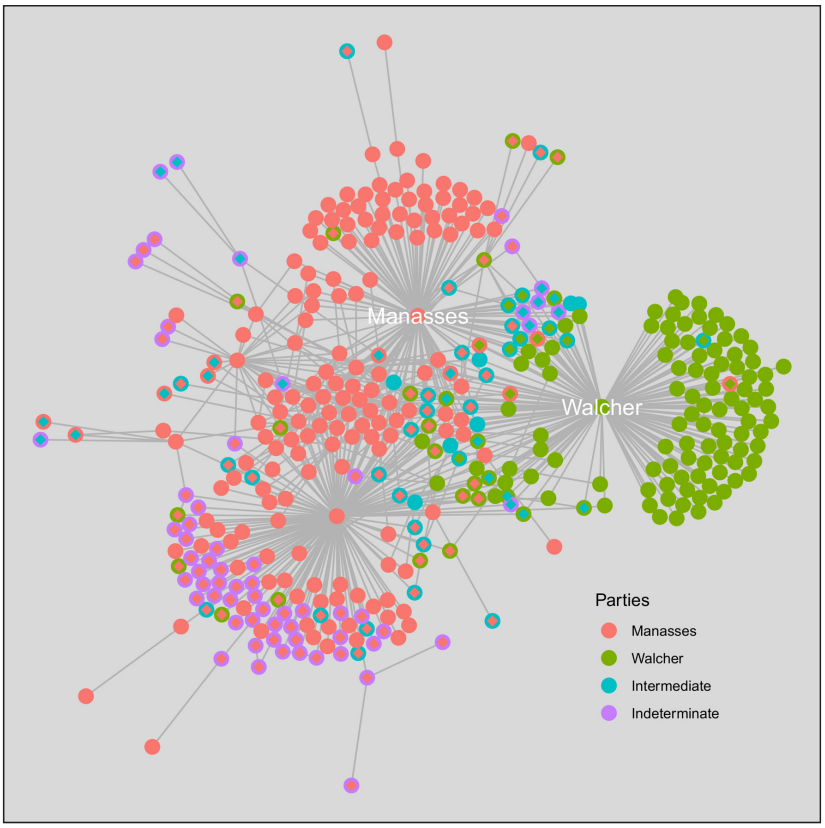

Figure 6: Comparison of the parties manually attributed (outer color) and the parties automatically attributed (inner color).

		automatically attributed parties		
		Manasses	**Walcher**	**Intermediate**
manually attributed parties	Manasses	159	4	7
	Walcher	18	102	7
	Intermediate	23	8	7

Table 2: Confusion matrix of the automatic reconstruction of the parties.

§39 Looking at the diagonal of this table yields the general accuracy of the method: for 80% of the vertices whose party is identified by the manual procedure (i.e. whose category is not "Indeterminate"), the two classifications correspond exactly.

§40 In this table most of the vertices that are wrongly assigned by the computer belong to the "Intermediate" party. This is not surprising, since this category is the most uncertain or at least the most difficult to make objective in manual attribution. In a few isolated cases, the sources explicitly mention that some person played a neutral role in the conflict (Van Mingroot, 1991). However, in other cases, the classification of a vertex in this group corresponds rather to the situation where an individual who, by virtue of his function, necessarily comes into contact with members of both parties. Therefore, the absence of a document in which he takes sides for one or the other leads us to categorize him as "Intermediate".

§41 Let us now turn to the results of the relative-importance-of-relations assessment. **Figure 7** presents, for each edge category, the values of the classification ratio defined in Section 3.3 when the coefficient of this category in the linear combination to aggregate the layers into a single-layer network varies between 0 and 100. By comparing these values with the value calculated on the original network (which corresponds to a coefficient equal to 1, the level of which is represented by a dotted line on the plot), we can deduce that our level of understanding of the conflict increases (higher value) or decreases (lower value) according to the importance given to the category in question. When the coefficient is smaller than 1, we observe what happens when we give this category less importance (or even simply remove it if the coefficient is zero). When, on the contrary, this coefficient is greater than 1, we observe what our understanding of conflict gains or loses when the importance of the category is inflated. Among the curves corresponding to the nine categories, four show a rather stable behavior, very close to the dotted horizontal line: Alliance, Consent, Kinship, and Notice. The temptation to conclude that the importance given to these types of relationships has no impact on the understanding that the historian can extract from

the network must be curbed. These edge types are the least represented in our graph (see **Table 1**), which leads us to conclude that their number is not sufficient to observe an effect on the metric.

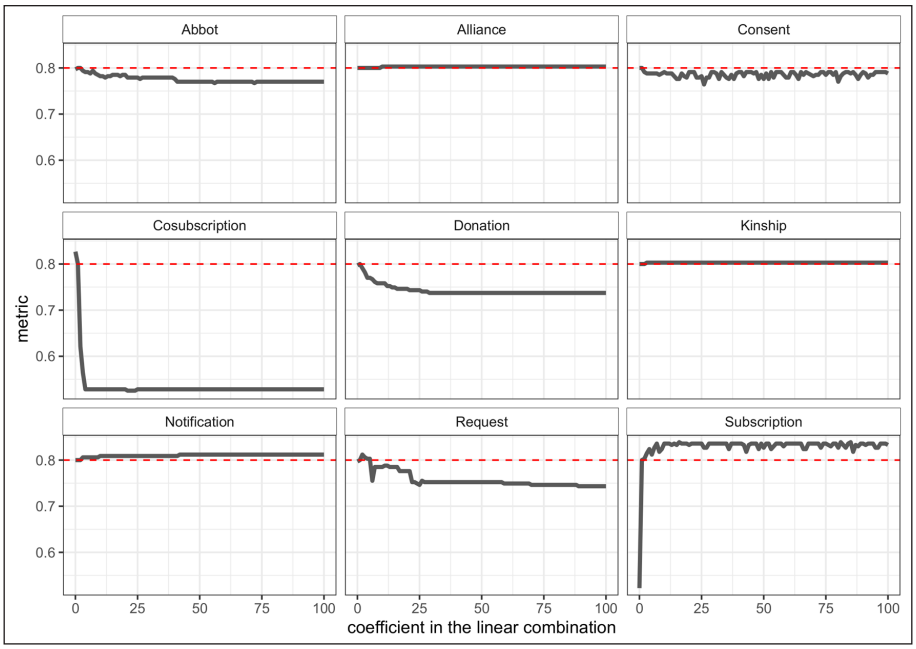

Figure 7: Evolution of the metric when increasing the importance of each of the nine relationships categories.

§42 A second group of relationship categories consists of Abbot, Donation, and Request, with mid-ranged numbers of attestations. For each of them, one observes a value very close to the dotted line when the coefficient is 0. Not considering these values therefore does not completely destroy the structure of political parties. In addition, values lower than the dotted line are observed when the coefficient is large. Inflating the importance of each of these three types of edges therefore lowers the level of understanding of the conflict.

§43 Note that the low significance of the seven categories analyzed so far does not mean that it is relevant to simply neglect them. The analysis we are conducting considers only the transformations carried out on one of the coefficients of the linear combination at a time (and thus one of the categories at a time).

§44 The case of the Subscription category, which is that of many network relationships, is completely different. The origin of the curve (at the extreme left of the plot) is well below the level of the dotted line: omitting these edges significantly deteriorates the party structure. The curve then rises above the reference value

when this category is given greater importance. Links of this type thus have a strong explanatory power within the graph. Understanding of conflict improves when they are considered at a larger extent than those of the Abbot, Donation, and Request categories. Note, however, that the curve stabilizes fairly quickly around a plateau value. This could indicate that, while inflating the importance of these edges improves the metric, it is not necessary to give them an absolutely central place. This could also suggest that the metric is reaching a "maximum" level that the limitations mentioned at the beginning of this section impose on it. These conclusions provide an answer to the first question posed in Section 1 of the paper.

§45 The analysis of the curve of the Cosubscription category makes it possible to take a very clear position on the role of these relations. Omitting them increases the value of the classification ratio, while inflating their importance causes it to plummet. We can thus formulate a rather clear-cut answer to the second question asked in Section 1: it is not relevant to encode cosubscription relations. Historians can therefore spare themselves this long and tedious work, since considering these edges, in addition to making the visual representations of the graph tangled and confused, deteriorates their understanding of the conflict. Note, however, that indirect relations of cosubscription are hidden in the relations of subscription. Indeed, since we consider the edges of the graph to be non-oriented, two vertices A and B, each connected to one of the candidates for the episcopal see, say Manasses, are also connected to each other through it. Even in the absence of cosubscription relations, there is therefore a path between A and B, but it passes through Manasses. Our results show that this indirect path is sufficient to account for the structuring of the network into parties.

§46 Finally, **Figure 8** shows the results obtained in the analysis of the order of individuals on the witness list. On this plot we can see the values of the classification ratio (thin light gray curve) when the alpha parameter takes values from 0 to 10 (see Section 3.4), as well as a smoothing of these values (thick dark gray curve). The diagnosis is irrevocable: the metric decreases as alpha increases. This means that widening the gap between the importance given to the first and last persons mentioned on the witness lists decreases our level of understanding of the conflict. All individuals on this list play a similar role in the network in terms of party structuring.

IV.2. Discussion

§47 The results presented in the previous section call for technical and historical comments, mainly concerning the working hypotheses used to obtain them. Let us begin with the technical ones, and first discuss the choice of the distance used in the automatic party-attribution procedure. To improve the robustness of our results, we

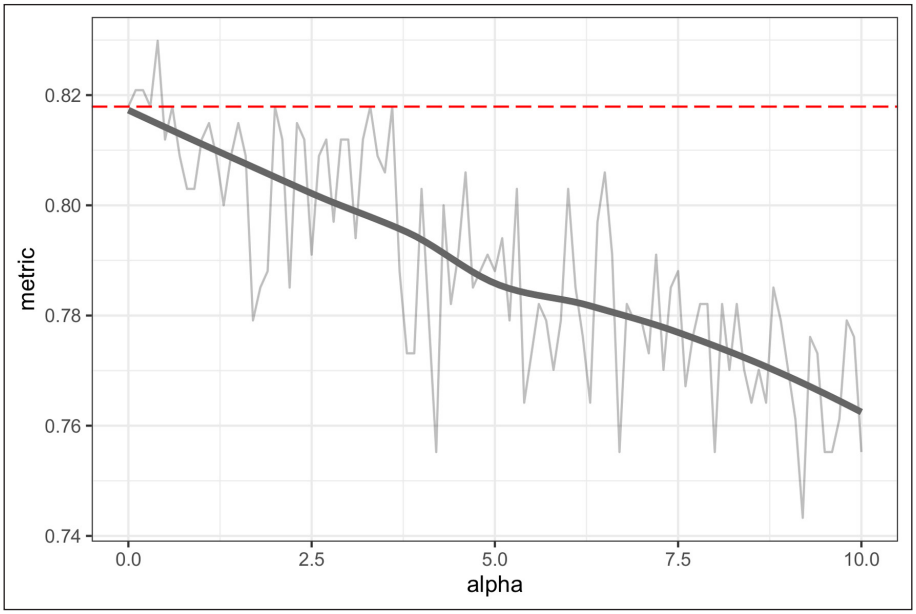

Figure 8: Evolution of the metric when increasing the parameter alpha (plus a trend curve).

performed all the calculations presented above twice: once with the distance *d* that we defined in Section 3.1, and once with a second distance *d'*. We have chosen the latter in order to take into account a "default" of *d*: only the minimum total cost path linking *i* and *j* is considered in the calculation of *d(i,j)*. The existence of other paths connecting *i* and *j* with higher total cost therefore has no impact on the value of *d(i,j)*. This characteristic of *d* is relatively counter-intuitive when considering a graph representing a network of social relations.

§48 This counter-intuitiveness is the reason why we have chosen the resistance distance for *d'*. This distance, which was introduced by Klein and Randic (1993), is based on a physical analogy (as the rigorous definition of this distance is rather technical, we will refer the reader to the original article for details). The graph is considered as an electrical network, in the edges of which electric current is passed. Each edge acts as a conducting wire whose resistance (in ohms) is equal to the cost: the higher the cost, the higher the resistance and the less easily the current passes. The resistance distance between *i* and *j* is then equal to the total effective resistance between the two vertices. The existence of multiple paths connecting *i* and *j* thus reduces the resistance distance. **Figure 9** compares the values of the two distances *d* and *d'* on several simple examples.

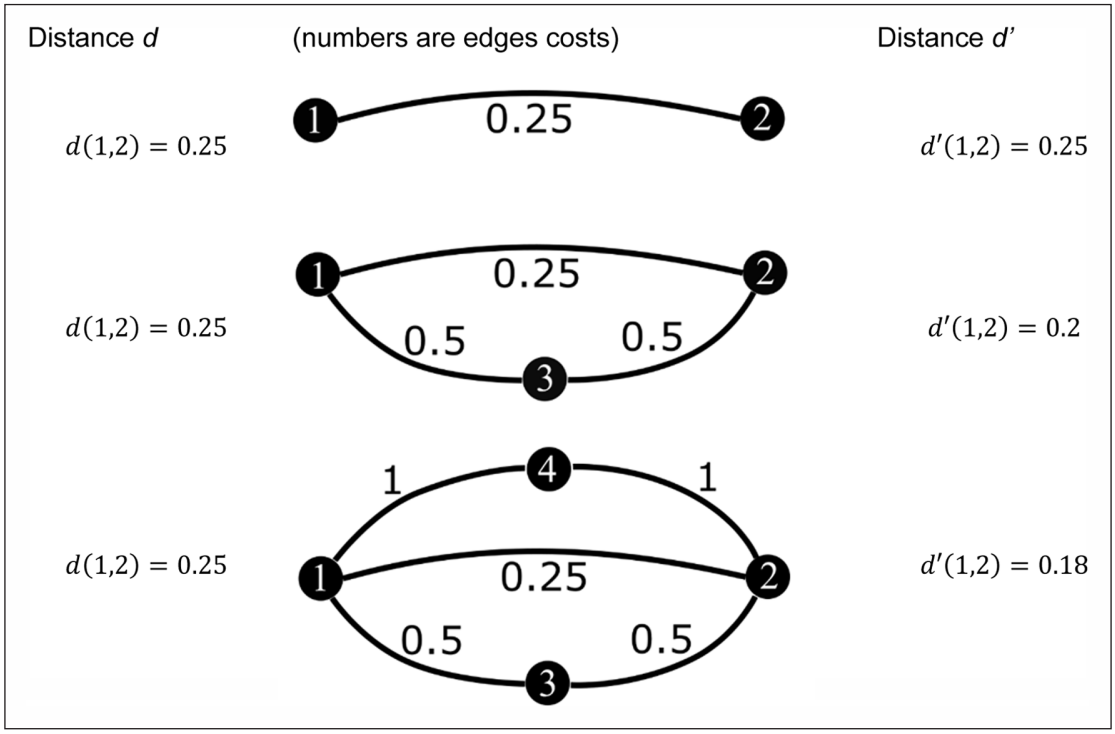

Figure 9: Comparison of distances *d* and *d'* on graph examples.

§49 All of the conclusions presented above were the same when distance *d'* was used. This shows that the default of the simple distance *d* used in our methodology does not impact our results.

§50 Let us nuance the interpretation of the metric we use to compare graphs. On the one hand, note that in some cases misattribution is inevitable, and does not depend on the design of the partitioning algorithm we have implemented, but rather on the overall approach of our study. This is, for example, the case of the vertex that, in **Figure 8**, appears to the right of Walcher and is colored green on the inside and red on the outside. The manual attribution therefore places it in the Manasses' party, unlike the automatic attribution that associates it with Walcher's. It could not be otherwise, since this vertex is directly and uniquely linked to the imperial candidate: no matter how the distances are calculated, it will be classified in Walcher's party. This structural obstacle is a direct consequence of the way we estimate the accuracy of automatic attributions, by comparing them to manual attribution. The metric is therefore subject to an upper bound, as it cannot reach the value of 100%.

§51 Let us now turn to historical comments about the method we implemented in this paper. The witness lists must be handled with care in social network analysis. In addition to the methodological difficulties already mentioned in the introduction, to which this article gives some answers, we should bear in mind that some lists are probably incomplete. We could not be sure that the list is complete, as the scribes might have decided to not write the names of all the witnesses present when the action was promulgated (Tock 1991). However, we could reasonably assume that the most prominent witnesses (princes, members of the high aristocracy, abbots, etc.) are mentioned in the witness lists, as it would be very surprising that the scribes would pass the names of the most powerful supporters of the bishops over in silence.

§52 Another difficulty in the use of the witness lists comes out of the reliability of the charter editions. Most of the Cambresian episcopal charters no longer exist in the original. Some disappeared before the French Revolution, others were destroyed in the bombing of the Belgian State Archives in Mons and Tournai during the Second World War. Consequently, a lot of these documents are only known through cartulary copies and early modern editions. We do not know to what degree these copies are faithful to the original charters. We cannot verify, for instance, whether the author of the copy included all witnesses in his document or whether he read their names correctly. Neither do we know whether he wrote their names in the correct order, as it is sometimes difficult to determine if a witness list is organized horizontally or vertically.

V. Conclusion

§53 Using a graph to study a historical dossier is a modelling activity. Like any model, the graph gives a simplified representation of the phenomenon it describes. In order to be useful and feasible, the applied simplifications for the analysis should neither be too much nor too little: it is a question of finding the right balance. In this paper, we looked at one aspect of historical network analysis touching on these simplifications. Historians using such tools are often confronted with sets of heterogeneous interpersonal relationships, which have to be brought together to build a network. The method for carrying out this gathering is a working hypothesis, the degree of simplification of which needs to be assessed. This is the analysis we have performed in this article, applied to a network modelling the Cambrai Investiture Contest.

§54 We have seen that the nine categories of edges of this graph are not all equally useful as to understand the structuring of its vertices into parties. Some of them improve

the metrics we have constructed to quantify this degree of understanding, others deteriorate it. Simplifications that just neglect the categories of links and consider them all equally should therefore be avoided.

§55 In particular, we have shown that subscription relationships play a prominent role within the graph. Special care must therefore be taken in the collection and encoding of these relations. They cannot be neglected and must be kept in case the graph is reduced (eg., for visualization purposes). Alongside this master category, the other types of relationships, which are generally related to the legal actions of the charters, look pale. Our study associates them with an invisible or slightly negative effect on the degree of understanding of the network structure.

§56 This hierarchy between the categories of edges of the graph also has historical implications that go beyond the methodological aspect, since it runs parallel to a hierarchy between the real-life interpersonal relations of the actors of the Cambrai Investiture Struggle. It shows that these types of relationships are not of equal importance for medieval people, at least in this precise chronological and geographical context. Our results suggest that acting as a witness for an authority was a much more committed action (here, in terms of choosing one side of the conflict) than entering with him into a legal action such as a confirmation, consent or donation. This is, in any case, the representation that medieval people have of it since, again, our methodology is based on a reconciliation of the information given in the charters with that reported in narrative sources.

§57 On the other hand, results about cosubscription relations show a significant negative impact: these edges do damage the picture of party formation we draw from the graph. They can (and should) therefore be omitted, at least when their number is large compared to edges of other types. In doing so, the historian avoids an encoding step that usually entails an important workload and does not risk unbalancing the analyses of the graph.

§58 This result may seem counter-intuitive, when one considers the idea of transitivity of "positive" interpersonal relationships to be natural. Indeed, one might expect that the fact that two individuals are connected to the same third individual would create an interpersonal link of some strength between them. It has been shown in this article that such a direct and full-strength transitivity is not observed for the relations associated with witness lists: integrating into the network cosubscription relations that make the subscription relations transitive does not improve the structure of the automatically retrieved parties. However, we cannot conclude that it is completely

absent from our network, since our working hypotheses (considering non-oriented graphs and our choice of distance) imply the existence of an indirect and relatively weak link between two witnesses on the same list.

§59 In the same line of thought, we also answered an issue historians have raised for more than fifty years. We have demonstrated that rank order in the witness lists is not crucial in social network analysis, at least during the high Middle Ages. This result does not mean that the rank of the witnesses in medieval charters was arbitrary. Rank order in witness lists had a social and a political meaning in the Middle Ages, as previous works have demonstrated. However, the rank order is not meaningful in the study of the structure of a network. In such circumstances, even poor editions of medieval charters, in which the order of the witnesses was not respected, may be used in social network analysis.

§60 The results of our study may be useful to historians using witness lists in social network analysis. Nevertheless, they must be confirmed by further inquiries devoted to earlier periods (for instance, the witness lists of the oldest French royal charters studied by Lemarignier in his seminal book of 1965) or from regions where diplomatic practices were different (for example, the ducal charters of Poitiers and Aquitaine examined by Prell 1997). Strengthening our results by further studies could change and improve our encoding practices, as the encoding of multi-faceted relationships in historical networks has raised a lot of critical questions, as Isabelle Rosé has underlined (Rosé 2011 and 2020).

§61 However, even though the result of our inquiry could improve the modelling practices of historians, it makes no doubt that a good understanding of a complex historical phenomenon relies on a quantitative approach as well as on the use of "classical" qualitative methods. These approaches are more complementary than antagonistic. In the case of the Investiture Contest in Cambrai, narrative sources such as the *Gesta Galcheri* help us to understand the global structure of the conflict and the sequence of events. However, the use of a quantitative approach mainly based on diplomatic sources, demonstrates the need to also qualify the accounts of the chronicles, for a combination of typologically different source material provides a more reliable picture of the parties at stake during the conflict. Contrary to what Bishop Gerard I–one of the most famous predecessors of Manasses and Walcher–wrote, the bishops of Cambrai and Arras did not "spend [their] years living in [their] city among the sword of the people of [their] country" (Bachrach, Bachrach, and Leese 2018) but could rely on a lot of allies from their own ecclesiastical entourage as well as from the secular world.

Competing Interests

The authors have no competing interests to declare.

Contributions

Authorial contributions

Authorship is alphabetical after the drafting author and principal technical lead. Author contributions, described using the CASRAI CredIT typology, are as follows:

Sébastien de Valeriola: SV

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All authors have contributed equally to this paper. The corresponding author is SV

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